ATTEMPT ALL QUESTIONS — MARKS

Total marks — 60

1. Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$.

(a) Evaluate $f(g(0))$. **1**

(b) Find an expression for $g(f(x))$. **2**

2. The point P $(-2, 1)$ lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$.

Find the equation of the tangent to the circle at P. **4**

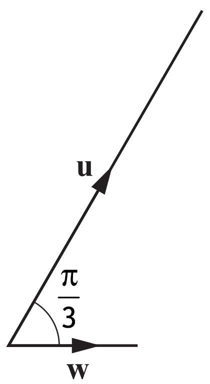
3. Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$. **2**

4. Find the value of k for which the equation $x^2 + 4x + (k-5) = 0$ has equal roots. **3**

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. **1**

(b)



Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}| = \sqrt{3}$.

Calculate $\mathbf{u}.\mathbf{w}$. **3**

MARKS

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$.

Determine an expression for $h^{-1}(x)$. **3**

7. A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.

Find the equation of the median through C. **3**

8. Calculate the rate of change of $d(t) = \dfrac{1}{2t}$, $t \neq 0$, when $t = 5$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

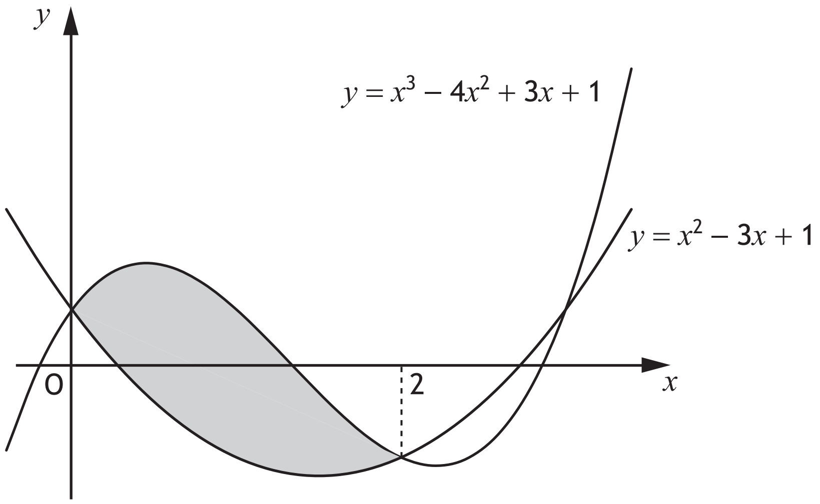
(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

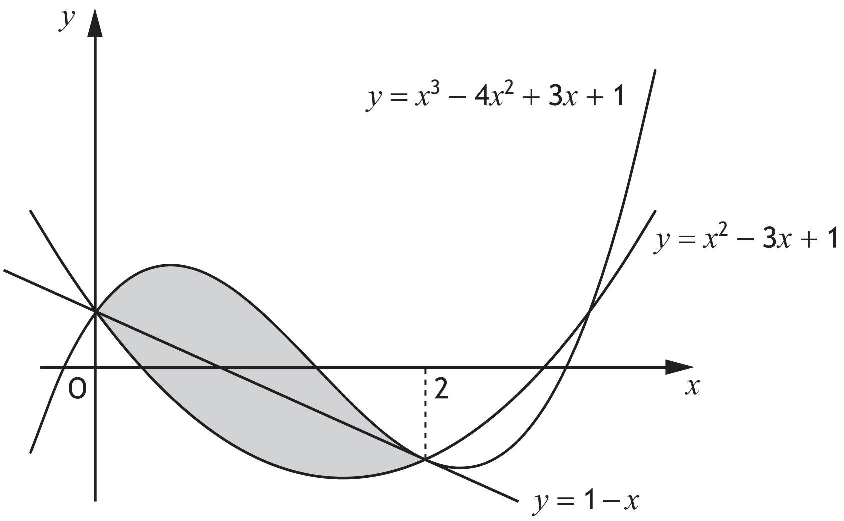
MARKS

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

[Turn over

MARKS

11. A and B are the points $(-7, 2)$ and $(5, a)$.

AB is parallel to the line with equation $3y - 2x = 4$.

Determine the value of a. **3**

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}} dx, \; x < \frac{5}{4}$. **4**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x - a)°$, where $k > 0$ and $0 < a < 360$. **4**

(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

Use the diagram provided in the answer booklet.

MARKS

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

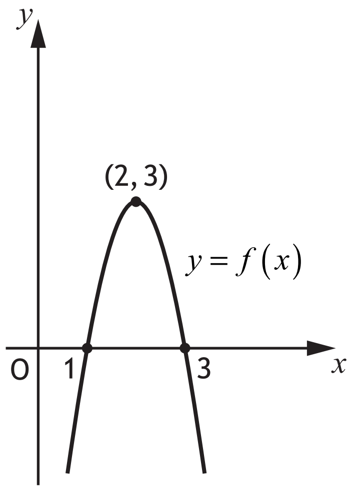


Diagram 1

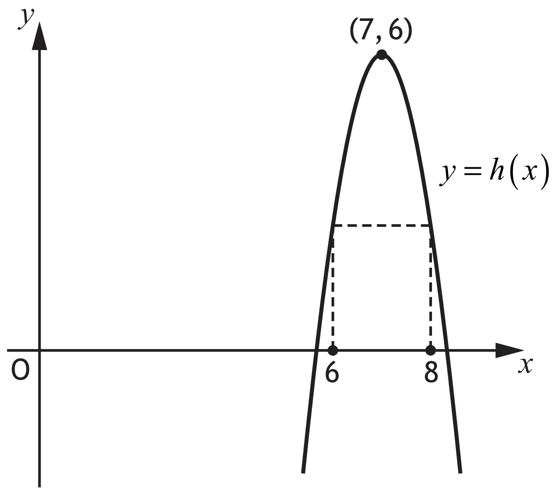


Diagram 2

(a) Given that $h(x) = f(x+a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

[END OF QUESTION PAPER]

MARKS

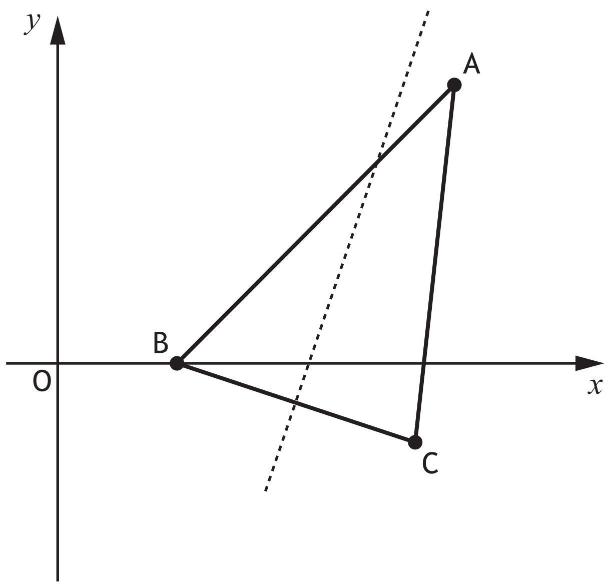
Attempt ALL questions

Total marks — 70

1. Triangle ABC is shown in the diagram below.

The coordinates of B are (3,0) and the coordinates of C are (9,–2).

The broken line is the perpendicular bisector of BC.



(a) Find the equation of the perpendicular bisector of BC. **4**

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.

Find the equation of AB. **2**

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. **2**

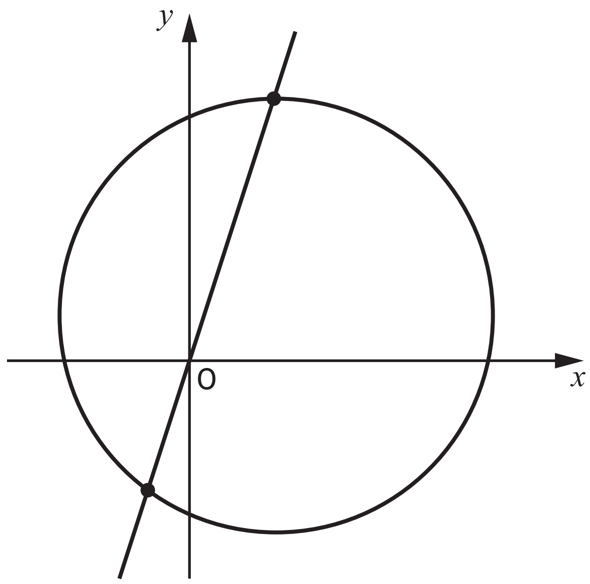
2. (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. **2**

(b) Hence, or otherwise, solve $f(x)=0$. **3**

[Turn over

MARKS

3. The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.



Find the coordinates of the points of intersection. **5**

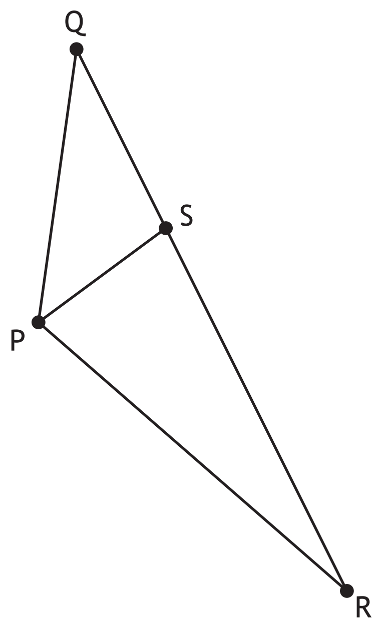
4. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. **3**

(b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x. **2**

MARKS

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **2**

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **2**

(c) Hence, find the size of angle QPS. **5**

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \leq x < 2\pi$. **5**

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **4**

(b) Hence, determine the greatest and least values of y in the interval $1 \leq x \leq 9$. **3**

[Turn over

MARKS

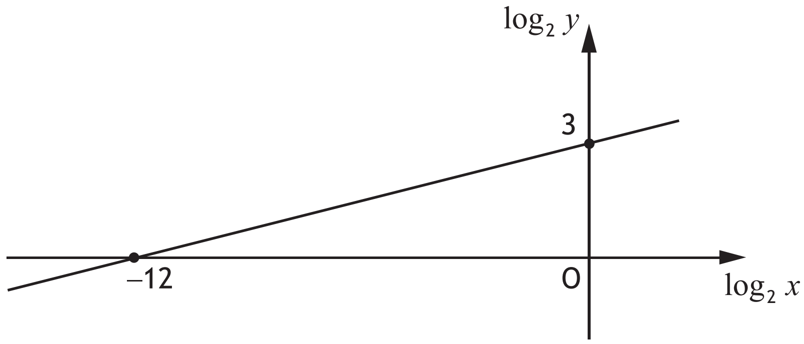
8. Sequences may be generated by recurrence relations of the form $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **2**

(b) Determine the range of values of k for which $u_2 < u_0$. **4**

9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.

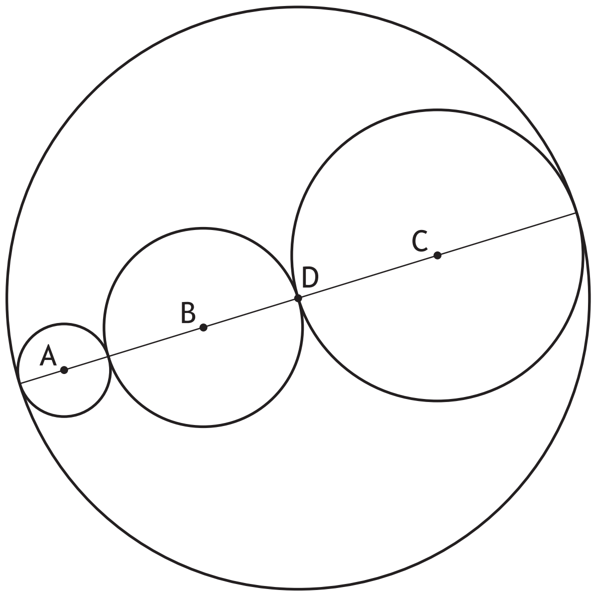


Find the values of k and n. **5**

MARKS

10. (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

11. (a) Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

(b) Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

[END OF QUESTION PAPER]